Question 7.1 :

A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Answer 7.1 :

Given :

The resistance R of the resistor is 100 Ω

The source voltage V is 220 V

The frequency of the supply is 50 Hz.

a) To determine the RMS value of the current in the connection, we use the following relation :

$$I = \frac{V}{R}$$

Substituting values, we get

$$I = \frac{220}{100} = 2.20 \text{ A}$$

Therefore, the RMS value of the current in the connection is 2.20 A.

b) The total power consumed over an entire cycle can be calculated using the following formula :

P = V x I

Substituting values in the above equation, we get

= 220 x 2.2 = 484 W

Therefore, the total power consumed is 484 W.

Question 7.2 :

a) The peak voltage of an ac supply is 300 V. What is the rms voltage?

b) The rms value of current in an ac circuit is 10 A. What is the peak current?

Answer 7.2 :

a) The peak voltage of the AC supply is V_0 = 300 V.

We know that,

$$V_{RMS} = V = \frac{V_0}{\sqrt{2}}$$

Substituting the values, we get

$$V = \frac{300}{\sqrt{2}} = 212.2 \text{ V}$$

The RMS voltage of the AC supply is 212.2 V.

b) The RMS value of the current in the circuit is I = 10 A

We can calculate the peak current from the following equation

$$I_0 = \sqrt{2}I$$

Substituting the values, we get

$$\sqrt{2} \times 10 = 14.1A$$

Question 7.3 :

A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.3:

As given :

The inductor connected to the AC supply has an inductance of $L = 44\ mH = 44 \times 10^{-3}\ H$

The magnitude of the source voltage V is 220 V

The frequency of the source is v = 50 Hz

The angular frequency of the source is given by $\omega = 2\pi v$

The Inductive reactance X_L can be calculated as follows:

$\omega_L = 2\pi v L = 2\pi \times 50 \times 44 \times 10^{-3}\ \Omega$

To find the RMS value of current we use the following relation:

à $I = \frac{V}{X_L} = \frac{220}{2\pi \times 50 \times 44 \times 10^{-3}}$ = 15.92 A

Therefore, the RMS value of the current in the network is 15.92 A.

Question 7.4 :

A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Answer 7.4 :

Given:

The capacitance of the capacitor in the circuit is $C = 60\ \mu F$ or $60 \times 10^{-6}\ F$

The source voltage is V = 110 V

The frequency of the source is v = 60 Hz

The angular frequency can be calculated using the following relation,

$\omega = 2\pi v$

The capacitive reactance in the circuit is calculated as follows:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\pi \times 60 \times 60 \times 10^{-6}}\ \Omega$$

Now, the RMS value of the current is determined as follows:

à $I = \frac{V}{X_C} = \frac{220}{2\pi \times 60 \times 60 \times 10^{-6}} = 2.49A$

Therefore, the RMS current is 2.49 A.

Question 7.5 :

In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer

Answer :

a) In the case of the inductive network, we know that

the RMS current value is I = 15.92 A

the RMS voltage value is V = 220 V

Therefore, the total power taken in can be derived by the following equation :

$P = VI \cos \Phi$

Here,

Φ is the phase difference between V and I

In case of a purely inductive circuit, the difference in the phase of an alternating voltage and an alternating current is 90°, i.e., Φ = 90°.

Therefore, P = 0

i.e., the total power absorbed by the circuit is zero.

b) In the case of the capacitive network, we know that

The value of RMS current is given by, I = 2.49 A

The value of RMS voltage is given by, V = 110 V

Thus, the total power absorbed is derived from the following equation :

P = VI Cos Φ

For a purely capacitive circuit, the phase difference between alternating Voltage and alternating current is 90°

i.e., Φ = 90°.

Thus , P = 0

i.e., the net power absorbed by the circuit is zero.

Question 7.6 :

Obtain the resonant frequency ω_r of a series LCR circuit with L = 2.0H, C = 32 μF and R = 10 Ω. What is the Q-value of this circuit?

Answer 7.6 :

Given:

The inductance of the inductor is L = 2.0 H

The capacitance of the capacitor, C = 32 μF = 32 × 10^{-6} F

The resistance of the resistor is R = 10 Ω.

We know that , resonant frequency can be calculated by the following relation,

à $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times32\times10^{-6}}} = \frac{1}{8\times10^{-3}} = 125\,\frac{rad}{sec}$

Now , Q – value of the circuit can be calculated as follows :

à $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32\times10^{-6}}} = \frac{1}{10\times4\times10^{-3}} = 25$

Thus , the Q – Value of the above question is 25.

Question 7.7 :

A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Answer 7.7 :

Given Capacitance value of the capacitor , C = 30 μF = 30 × 10^{-6} F

Given Inductance value of the charged inductor, L = 27 mH = 27 × 10^{-3} H

Angular frequency is given as :

à $\omega_r = \frac{1}{\sqrt{LC}}$

à $\omega_r = \frac{1}{\sqrt{27\times10^{-3}\times30\times10^{-6}}} = \frac{1}{9\times10^{-4}} = 1.11 \times 10^{3}\,\frac{rad}{sec}$

Therefore , the calculated angular frequency of free oscillation of the connection is 1.11 x 10^{3} rad / s

Question 7.8 :

Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

Answer 7.8:

Given Capacitance value of the capacitor, C = 30 μF = 30 × 10^{-6} F

Inductance of the inductor, L = 27 mH = 27×10^{-3} H

Charge on the capacitor, Q = 6 mC = 6×10^{-3} C

Total energy stored in the capacitor can be calculated as :

à $E = \frac{1}{2} \times \frac{Q^2}{C} = \frac{1}{2}\frac{\left(6\times10^{-3}\right)^2}{30\times10^{-6}} = \frac{6}{10} = 0.6J$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

Question 7.9:

A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Answer 7.9 :

The supply frequency and the natural frequency are equal at resonance condition in the circuit.

Given Resistance of the resistor, R = 20 Ω

Given Inductance of the inductor, L = 1.5 H

Given Capacitance of the capacitor , C = 35 μF = 30×10^{-6} F

An AC source with a voltage of V = 200 V is connected to the LCR circuit,

We know that the Impedance of the above combination can be calculated by the following relation,

à $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonant condition in the circuit , $X_L = X_C$

Therefore , Z = R = 20 Ω

We know that Current in the network is given by the relation :

à $I = \frac{V}{Z} = \frac{200}{20} = 10A$

Therefore , the average power that is being transferred to the circuit in one full cycle :

V I = 200 x 10 = 2000 W

Question 7.10:

A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[Hint: The condition for tuning is that the natural frequency which is the frequency of free oscillations of the LC network must be of the same value as the radio wave frequency]

Answer 7.10:

Given the frequency range of (v) of radio is 800 kHz to 1200 kHz.

Given that the Lower tuning frequency of the circuit is , v 1 = 800 kHz = 800 × 10 3 Hz

Given that the Upper tuning frequency of the circuit is, v 2 = 1200 kHz = 1200 × 10 3 Hz

Given that the Effective inductance of the inductor in the circuit is L = 200 μH = 200×10^{-6} H

We know that, Capacitance of variable capacitor for v 1 can be calculated as follows :

à $C_1 = \frac{1}{\omega_1^2 L}$

Here the variables are ,

ω_1 = Angular frequency for capacitor C_1

à $\omega_1 = 2\pi v_1$

à $\omega_1 = 2\pi \times 800 \times 10^3$ rad/s

therefore,

$$C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}} = 1.9809 \times 10^{-10} F = 198pF$$

Variable capacitor for v 2 has capacitance of :

$$C_2 = \frac{1}{\omega_2^2 L}$$

Here the variables are ,

ω_2 = Angular frequency for capacitor C_2

à $\omega_2 = 2\pi v_2$

à $\omega_2 = 2\pi \times 1200 \times 10^3$ rad/s

Therefore,

$$C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}} = 0.8804 \times 10^{-10} F = 88pF$$

Thus , the variable capacitor has a range from 88.04 pF to 198.1 pF.

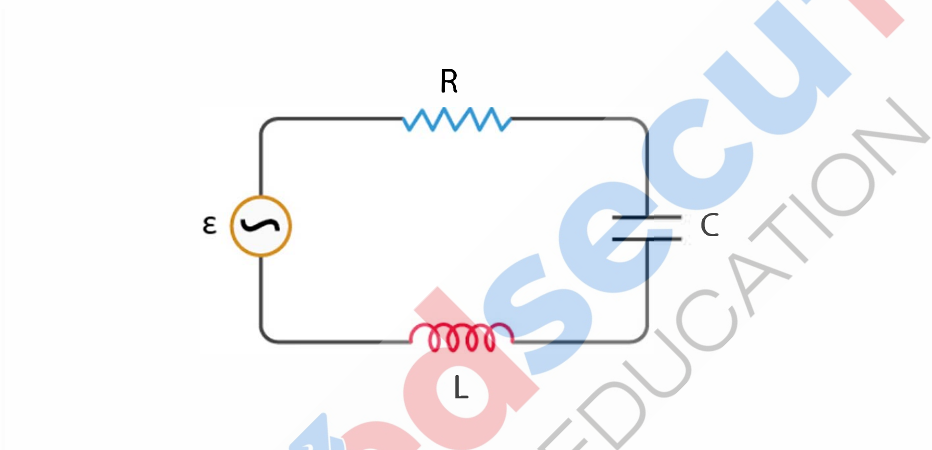
Question 7.11:

Figure below shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω.

L = 5.0 H,

C = 80 μF,

R = 40 Ω



(a) Determine the source frequency which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the RMS potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

Answer 7.11 :

Given that the Inductance of the inductor in the circuit is , L = 5.0 H

Given that the Capacitance of the capacitor in the circuit is , C = 80 μH = 80 × 10^{-6} F

Given that Resistance of the resistor in the circuit , R = 40 Ω

Value of Potential of the variable voltage supply, V = 230 V

(a) We know that the Resonance angular frequency can be obtained by the following relation :

à $\omega_r = \frac{1}{\sqrt{LC}}$ $\omega_r = \frac{1}{\sqrt{5x80x10-6}}$ $\omega_r = \frac{10^3}{20} = 50rad/sec$

Thus, the circuit encounters resonance at a frequency of 50 rad/s.

(b) We know that the Impedance of the circuit can be calculated by the following relation :

à $Z = \sqrt{R^2 + (X_L - X_C)^2}$

At resonant condition ,

$X_L = X_C$

Z = R = 40 Ω

At resonating frequency amplitude of the current can be given by the following relation :

à $I_0 = \frac{V_0}{Z}$

Where,

à V_0 = peak voltage = $\sqrt{2}V$

Therefore,

à $I_0 = \frac{\sqrt{2}V}{Z} = \frac{\sqrt{2}\times 230}{40} = 8.13A$

Thus, at resonant condition , the impedance of the circuit is calculated to be 40 Ω and the amplitude of the current is found to be 8.13 A

c) rms potential drop across the inductor in the circuit ,

à $(V_L)_{rms} = I \times \omega_r L$

Where,

$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{\sqrt{2}V}{\sqrt{2}Z} = \frac{230}{40} = \frac{23}{4}A$

Therefore, $(V_L)_{rms}$

$\frac{23}{4} \times 50 \times 5 = 1437.5V$

We know that the Potential drop across the capacitor can be calculated with the following relation :

à $(V_c)_{rms} = I \times \frac{1}{\omega_r C} = \frac{23}{4} \times \frac{1}{50\times 80\times 10^{-6}} = 1437.5V$

We know that the Potential drop across the resistor can be calculated with the following relation :

à $(V_R)_{rms} = IR = \frac{23}{4} \times 40 = 230V$

Now, Potential drop across the LC connection can be obtained by the following relation :

à $V_{LC} = I(X_L - X_C)$

At resonant condition ,

à $X_L = X_C$

à $V_{LC} = 0$

Therefore, it has been proved from the above equation that the potential drop across the LC connection is equal to zero at a frequency at which resonance occurs.